Can Quantum-Mechanical Description of Reality be Considered Complete?

A. Einstein, B. Podolsky, and N. Rosen

Received March 25, 1935

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1

Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one: every element of the physical reality must
have a counterpart in the physical theory. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by a priori philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one such way, whenever the conditions set down in it occur. Regarded not as a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.

...it is shown in quantum mechanics that, if the operators corresponding to two physical quantities, say $A$ and $B$, do not commute, that is, if $AB \neq BA$, then the precise knowledge of one of them precludes such a knowledge of the other. Furthermore, any attempt to determine the latter experimentally will alter the state of the system in such a way as to destroy the knowledge of the first.

From this follows that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. For if both of them had simultaneous reality—and thus definite values—these values would enter into the complete description, according to the condition of completeness. If then the wave function provided such a complete description of reality, it would contain these values; these would then be predictable. This not being the case, we are left with the alternatives stated.

In quantum mechanics it is usually assumed that the wave function does contain a complete description of the physical reality of the system in the state to which it corresponds. At first sight this assumption is entirely reasonable, for the information obtainable from a wave function seems to correspond exactly to what can be measured without altering the state of the system. We shall show, however, that this assumption, together with the criterion of reality given above, leads to a contradiction.

2

For this purpose let us suppose that we have two systems, I and II, [and] let us designate the corresponding wave function by $\Psi$. ...
where the $\psi_n$ are vectors used to describe the second system. Suppose now that the quantity $A$ is measured and it is found that it has the value $a_k$. It is then concluded that after the measurement the first system is left in the state given by the vector $u_k$, and that the second system is left in the state given by the vector $\psi_k$. This is the process of reduction of the wave packet; the vector given by the infinite series (7) is reduced to a single term $u_k \otimes \psi_k$.

The set of vectors $u_n$ is determined by the choice of the physical quantity $A$. If, instead of this, we had chosen another quantity, say $B$, having the eigenvalues $b_1, b_2, b_3, \cdots$ and eigenvectors $v_1, v_2, v_3, \cdots$ we should have obtained, instead of Eq. (7), the expansion

$$\Psi = \sum_n \phi_n \otimes v_n \quad (8)$$

If now the quantity $B$ is measured and is found to have the value $b_r$, we conclude that after the measurement the first system is left in the state given by $v_r$ and the second system is left in the state given by $\phi_r$.

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different state vectors. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus, it is possible to assign two different wave functions (in our example $\psi_k$ and $\phi_r$) to the same reality (the second system after the interaction with the first).

Now, it may happen that the two state vectors, $\psi_k$ and $\phi_r$, are eigenvectors of two noncommuting operators corresponding to some physical quantities $P$ and $Q$, respectively. That this may actually be the case can best be shown by an example. Let us suppose that the two systems are two particles, and that in the z-spin basis,

$$\Psi = \frac{1}{\sqrt{2}} \left( \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \otimes \left( \begin{array}{c} 0 \\ 1 \end{array} \right) - \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \otimes \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \right) \quad (9)$$

[If $A$ is the z-spin of the first particle, then the corresponding $\psi_k$ must be an eigenvector of the z-spin of the second particle. If $B$ is the x-spin of the first particle, then the corresponding $\phi_r$ must be an eigenvector of the x-spin of the second particle. Hence, we have shown that it is in general possible for $\psi_k$ and $\phi_r$ to be eigenvectors of two noncommuting operators, corresponding to physical quantities.

Returning now to the general case contemplated in Eqs. (7) and (8), we assume that $\psi_k$ and $\phi_r$ are indeed eigenvectors of some noncommuting operators $P$ and $Q$, corresponding to the eigenvalues $p_k$ and $q_r$, respectively. Thus, by measuring either $A$ or $B$ we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity $P$ (that is $p_k$) or the value of the quantity $Q$ (that is $q_r$). In accordance with our criterion of reality, in the first case we must consider the quantity $P$ as being an element of reality, in the second case the
quantity $Q$ is an element of reality. But, as we have seen, both wave functions $\psi_k$ and $\phi_r$ belong to the same reality.

Previously we proved that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.

One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted. On this point of view, since either one or the other, but not both simultaneously, of the quantities $P$ and $Q$ can be predicted, they are not simultaneously real. This makes the reality of $P$ and $Q$ depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.

While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

Questions

1. “According to EPR, if every element of physical reality has a counterpart in the theory, then the theory is complete.” True or false?

2. What is EPR’s criterion of reality?

3. Make an argument map of the argument in paragraph (A).

4. Make an argument map of the argument in paragraph (B).

5. Express the state $\Psi$ (Eq. 9) in terms of x-spin eigenvectors. Use this to satisfy yourself that if the x-spin of the first particle is measured, the post-measurement state of the second particle must be an eigenvector of x-spin.

6. Reconstruct the logical structure of the paragraph labelled (C).

7. Make an argument map of the paper as a whole.