Week 10: Relativistic effects
Introduction to philosophy of physics: space and time
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Relativity of simultaneity

• In any classical spacetime, there is a unique and absolute standard of simultaneity

• In Minkowski spacetime, any parallel set of spacelike hyperplanes is as appropriate a standard of simultaneity as any other

• For any given observer (at a particular time), there is a naturally associated hyperplane: that which is perpendicular to the (timelike) curve they are following

• It follows that for any inertial observer, there is a natural standard of simultaneity to choose: the hyperplanes which are perpendicular to their trajectory at all times

• If two observers are in relative motion, then the natural standards they choose will not be the same

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Relativity of duration

- For any two points in any classical spacetime, there is a unique and absolute duration between the two points.

- For any two points $x, y$ in Minkowski spacetime such that $(y - x)$ is timelike, there is a uniquely privileged “duration” between them: the number $I(x, y)$.

- This is the duration which would be recorded by a clock that followed the inertial (straight) trajectory between $x$ and $y$.

- However, an observer $O$ will regard the duration between $x$ and $y$ as being given by $I(x^O, y^O)$, where $x^O$ is the point on the observer’s trajectory simultaneous with $x$, and $y^O$ is the point on the observer’s trajectory simultaneous with $y$.

- Assuming that $O$ is inertial, and that $O$ employs the natural standard of simultaneity (for her) discussed above, then $I(x^O, y^O) = I(x, y)$ only if the observer’s trajectory is at rest relative to the inertial trajectory from $x$ to $y$: otherwise, if the relative speed of the trajectories is $v$, then $I(x^O, y^O) = \gamma(v)I(x, y)$.

- Similarly, given another inertial observer $O’$ who employs his natural standard of simultaneity, then $I(x^{O’}, y^{O’}) = I(x^O, y^O)$ only if $O$ and $O’$ are at rest relative to one another.

Relativity of length

- For a pair of co-moving inertial trajectories through a classical spacetime (say, the trajectories of two ends of a rigid rod), every point on one trajectory is uniquely and absolutely associated with a point on the other via the unique and absolute simultaneity relation, and the distance between them gives the length of the rod.

- For such a pair of trajectories through Minkowski spacetime, there is a naturally privileged “length” $L$: the number $-iI(p, q)$, for any point $p$ on one trajectory and the point $q$ on the other which is simultaneous with $p$ according to the rod’s natural simultaneity relation.

- However, an observer $O$ will only regard $-iI(p, q)$ as the length of the rod if $p$ and $q$ are simultaneous (according to their chosen standard of simultaneity): otherwise, they will regard $p$ as simultaneous with some point $q^O \neq q$, and will regard the rod as being of length $L^O = -iI(p, q^O)$.

- If $O$ is moving inertially in the direction of the rod at speed $v$ relative to it, and that $O$ employs her natural standard of simultaneity, then $L^O = L/\gamma(v)$.

- Again, if $O$ and $O’$ are both inertial observers, employing their natural respective standards of simultaneity, then they will agree on the length of the stick at all times only if they are relatively at rest.